Cost of Capital when Dividends are Deductible
(Custo de Capital quando os Dividendos são Dedutíveis)

Ignacio Velez-Pareja*
Julián Benavides-Franco**

Abstract

Tax savings and the discount rate we use to calculate their value are involved in the calculation of cost of capital. Based on previous findings, we derive a general approach to cash flow valuation that take into account any kind of tax shields related to the financing decision of a firm and any date when they are earned. They can be used to introduce any type of externality that creates value through tax savings not captured by neither the cost of debt nor the cost of equity. This paper develops the formulations for the cost of capital when dividends, interest on equity or monetary correction of equity are deductible as it happens in Brazil. It shows that when properly done most known valuation methods are consistent and give identical results. Also, the paper argues that when dividends are tax deductible, optimal leverage is lower and equity value is higher.

***Keywords*:** corporate finance; *WACC*; interest on equity; tax savings; tax shields; cost of equity; discount rate for tax savings.

JEL codes: D61; G31; H43.

Resumo

As economias tributárias e a taxa de desconto que usamos para calcular o seu valor estão envolvidos no cálculo do custo de capital. Com base em resultados anteriores, nós derivamos uma abordagem geral para avaliação de fluxo de caixa que considera qualquer tipo de benefícios fiscais relacionados com a decisão de financiamento de uma empresa e qualquer data em que sejam auferidos. Eles podem ser usados para introduzir qualquer tipo de externalidade que cria valor por meio de economias tributárias não capturadas nem pelo custo da dívida nem pelo custo do capital próprio. Particularmente, nós desenvolvemos as formulações para o custo de capital quando os dividendos, juros sobre capital próprio ou correção monetária de capital próprio são dedutíveis, como acontece no Brasil. Isso mostra que, quando usados corretamente, os métodos de avaliação mais conhecidos são consistentes e dão resultados idênticos. Além disso, o artigo argumenta que quando os dividendos são dedutíveis, a alavancagem ótima é menor e o valor do capital acionário é maior.

***Palavras-chave*:** finanças corporativas; *WACC*; juros sobre capital próprio; economias tributárias; benefícios tributários; custo do capital próprio; taxa de desconto para economias tributárias.

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*Universidad Tecnológica de Bolivar, Cartagena, Bolivar, Colombia. E-mail: nachovelez@gmail.com

**Universidad Icesi, Cali, Valle. Colombia. E-mail: jbenavid@icesi.edu.co

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1. Introduction

Since the seminal contribution of Modigliani & Miller (1963) the value of Tax Savings, TS, has been recognized by the literature. Most of the analyses, including Modigliani & Miller (1963), have focused on the Tax Savings of Debt. One exception is DeAngelo & Masulis (1980) that consider TS due to investment incentives and depreciation. When calculating Tax Savings, TS, we are confronted with a strange mix of accounting accrual and market value when involving TS in the calculation of the Weighted Average Cost of Capital, $WACC$ or the Cost of Equity, Ke. Firms earn the right to TS once they accrue the interest expense and they actually earn the TS when taxes are paid (Vélez-Pareja *et al.*, 2008).

Tax savings and the discount rate (ψ) we use to calculate their value are involved in the calculation of $WACC$ and Ke. Textbook $WACC$ formulation is a very special and unique case that is not typical. Based on previous findings, we derive a general approach to those formulas that take into account any kind of TS related to the financing decision of a firm and any date when the TS is earned. These formulations can be used to introduce any type of externality that creates value through tax savings not captured by neither the cost of debt nor the cost of equity.

Taggart (1991) developed some expressions with these purposes. He considers corporate and personal taxes; we only consider corporate taxes. There is no derivation of the formulas in his work. Inselbag & Kaufold (1997), use the same expressions to compare different discount methods to value a firm. Tham & Vélez-Pareja (2002) and Tham & Vélez-Pareja (2004) derive the proper formulations for Ke and $WACC$. Vélez-Pareja (2010) use these derivations to incorporate the effect of losses in exchange rate, of losses carried forward, of unpaid taxes, of Presumptive Income Taxation and the effect of inflation adjustment of book value of equity when adjusting financial statements by inflation in tax savings. We derive the explicit and general formulation to include other sources of TS and their discount rate, ψ.

These refinements for calculating Ke and $WACC$ are based on Modigliani & Miller propositions and they are just a tuning up of them to include idiosyncratic conditions found in different markets. In this paper we study the impact of any new source of tax savings. In particular, we study the specific case of Brazil, one of the major economies in the world, which allows a partial deduction of dividends in the Profit and Loss statement. Some other countries also allow (or allowed) the partial deduction of dividends, such as Iceland, Czech Republic and Germany. Although the tax deduction we consider here is novel, and absent in previous works, the basic ideas posed by M&M are the same. However, when valuing firms or projects in such environments (e.g. Brazil, Ireland, etc.) our model captures a source of value previously ignored.

When Brazil used to adjust the financial statements by inflation, they (as many other economies) allowed for adjustment of book value of equity using an index linked to the inflation rate. According to Zani & Ness (2001) after many years of inflation adjustment with a charge equal to the adjustment of book value of equity, since January 1, 1996 firms were allowed to charge interest on the book value of equity and had not only the effect to be a deductible charge, but to pay those interest expenses as part of the dividends defined by the firm.[1] What initially was an accounting accrual figure now is now an actual payment to shareholders with the associated tax savings benefits as before.

In the financial report of a Brazilian firm, Aracruz Celulose[2] to the U.S. Securities and Exchange Commission, they say:

> As of January 1, 1996, Brazilian corporations are allowed to attribute interest on stockholders' equity. The calculation is based on the stockholders' equity amounts as stated in the statutory accounting records and the interest rate applied may not exceed the long-term interest rate ("TJLP") determined by the Brazilian Central Bank (approximately 9.75%, 7.78% and 6.32% for years 2005, 2006 and 2007, respectively). Also, such interest may not exceed the greater of 50% of net income for the year or 50% of retained earnings plus income reserves (including those mentioned above), determined in each case on the basis of the statutory financial statements. The amount of interest attributed to stockholders is deductible for corporate income tax purposes.[3]

Non-traded stock corporations may pay interest on equity JSCP by its initials in Portuguese. The long term interest rate ("A Taxa de Juros de Longo Prazo – TJLP" in Portuguese) is not a market rate. It is established by the National Monetary Council (Conselho Monetário Nacional) and used for loans by the BNDES. "The Brazilian Development Bank (BNDES) is a federal public company, linked to the Ministry of Development, Industry and Foreign Trade (MDIC). Its goal is to provide long-term financing aimed at enhancing Brazil's development, and, therefore, improving the competitiveness of the Brazilian economy and the standard of living of the Brazilian population."[4]

This practice, apart from the adjustment for inflation that was made on an accrual basis, is unusual in the sense of becoming an actual payment, a cash flow. This is not a new cash flow, but it is part of the dividends defined by the firm and yet, they are deductible and hence, the firm earns TS on that. Although the deduction was created as a compensation for the mainly negative result of the inflation adjustment in the income statement, it had unanticipated consequences on the valuation of firms. Damodaran (2003) studies a proposal similar in spirit to the one

[1] Act 9,249/95, Article 10.

[2] Acquired in 2009 by Votorantim Celulose e Papel, now Fibria, listed in the Novo Mercado of BM&BOVESPA.

[3] `http://www.aracruz.com/minisites/ra2005/localaracruz/ra2005/en/if/demonstracoes_notas.html`, visited on June 14, 2009.

[4] `http://www.bndes.gov.br/english/thecompany.asp` visited June 15, 2009.

we study. In 2003 the Bush Administration in its economic package, proposed that the full amount of dividends would be deductible, in order to eliminate double taxation of dividends. By allowing corporations to deduct its dividend payments, taxation would be assumed by the shareholders. Additionally to discuss the effect on cash flows and discount rates, Damodaran speculates that this change would induce firms to be more equity financed (if not entirely), to reduce their cash balances and pay more dividends.[5]

Our results point out that this regulation should have empirical consequences similar to those envisioned by Damodaran (2003). Firms working in such environment are more valuable for their owners and should be less leveraged, as a simple trade-off model proves it. While empirical analyses are in order, the macro economic consequences of such regime are also worth of discussion. As the appetite for debt is reduced, the costs of financial distress will also fall, reducing the economic impact of failed firms for the society, while at the same time increasing the propensity for new ventures, by lowering the risk assumed by the owners.

Different authors have tackled non debt deductions, DeAngelo & Masulis (1980), take Miller's irrelevance model (1977) and postulate that the existence of debt-unrelated tax shields, such as investment tax benefits or depreciation, create optimum financing conditions, even without considering debt-related bankruptcy costs. The existence of these shields reduces the optimal level of financial leverage. Our analysis, considering bankruptcy costs, reaches a similar conclusion when a dividend tax shield is allowed. Graham & Tucker (2006) study US cases where firms engaged in allegedly illegal practices, they document a reduction in financial leverage when companies make use of tax shelters and report fictitious losses.

In a different venue, Rao & Stevens (2007) argument that previous analyses on capital structure ignore the third claim on the firm cash flows, concentrating on the creditors' and shareholders' claim. They develop a model which also values the government claim on the firm, shedding light on the appropriate rate of discount for a firm's debt, and the different tax shields a firm can have. They value non-debt tax shields (depreciation) using, what they call the approximate APT and contend that the appropriate discount rate for the debt is Kd.[6]

The work closest to our own is Zani & Ness (2001) that consider the effect of deductible dividends ("Juros sobre o capital próprio") on Brazilian firms' leverage. They modify the Miller's (1977) irrelevance model with personal taxes to include two additional terms, the extra cash flow received by the dividend tax shield and the bankruptcy costs. Assuming perpetual and constant cash flows and that the appropriate discount rate for the dividend tax shield is the unlevered cost of equity (Ku, per our terminology, see section 2), they calculate the extra value the share-

[5] The American Congress finally approved a reduction of the tax on dividends to 15%. See Section 4.

[6] Rao & Stevens (2007) focus their work on finding appropriate discount rates for the firm cash flows, by valuing the third claim (taxes) on the firm, absent in previous analyses. Our job focus in developing a working set of equations for valuing purposes, including additional tax shields, absent in previous models.

holders get because the deductible dividends and a modified expression to evaluate the debt advantage for Brazilian firms. Under the tax rates of 1998, they find that the attractiveness of debt is significantly reduced after the introduction of the new regulation. Their empirical tests, however, do not find evidence of a reduction in leverage after some Brazilian firms initiated the payment of deductible dividends (JSCP) even though its fiscal burden was reduced. Our work complements Zani & Ness (2001) work in two important ways. First, we develop a workable set of equations for valuation purposes that not rely in perpetual and constant cash flows; neither have we assumed that the appropriate rate of discount for the deductible dividends is the unlevered cost of equity. Second, we show a general expression for the debt tax shield for perpetual and constant cash flows that includes personal taxes and the adjustment for deductible dividends.

The rest of the paper is organized as follows. Section 2 introduces the framework and develops the valuation equations. Section 3 values a hypothetical cash flow, shows the coherence of the model. Section 4 solves a modified trade-off model. Section 5 concludes.

2. Framework

Our framework begins outlining the income statement that allows for the deductibility of dividends. Second, we define our variables and develop the valuation equations for the cash flow to equity, the free cash flow and the capital cash flow.

The Income Statement

An Income Statement according to the Brazilian regulation would be as follows:

Table 1
Income Statement with Dividends Deducted as an Expense

Earnings Before Interest and Taxes	EBIT
- Interest on Debt	$-K_d * D$
- Interest on book value of equity	$-K_{f'} * E$
= Earnings Before Tax	$= EBT$
- Income taxes	$-T * EBT$
= Net Income	$= NI$
- Dividends	- Dividends paid
= To Retained Earnings	= Add to Retained Earnings

The financing cash flow can be disaggregated as follows:

- Cash Flow to Equity, CFE

$$\begin{aligned} CFE &= EBIT(1-T) - WC - Kd \times D \times (1-T) \\ &+ D + K_{f'} \times E \times T \\ CFE &= FCF - CFD + TS^D + TS^E \end{aligned} \quad (1)$$

- Cash Flow to Debt, CFD

$$K_d \times D - D = CFD \quad (2)$$

- Capital Cash Flow, CCF

$$FCF + TS^D + TS^E = FCF + Kd \times D \times T + K_{f'} \times BVE \times T \quad (3)$$

It is clear that the CFE is increased by TS_{equity} and this fact has to be included in the derivation of Ke and $WACC$.

Variables in equations

$WACC_{gen}$ = Weighted Average Cost of Capital in a general formulation;
Rm = Market return;
Rf = Risk free rate;
MRP = Market Risk Premium = $Rm - Rf$;
Ku = Cost of unlevered equity and can be calculated using CAPM or any other procedure;
$Ku = Rf + u \times (Rm - Rf)$;
Ke = Cost of levered equity;
E = Discount rate for tax savings from equity interest;
D = Discount rate for tax savings from debt interest;
Kd = Cost of debt;
T= Corporate Tax Rate;
FCF = Free Cash Flow;
$K_f^{'}$ = Interest rate on book equity.

General Formulations for Ke, WACC for the FCF and for the CCF

In this Section we develop the formulations for the cost of capital taking into account the tax savings when interest on equity (or dividends) is deductible. The derivation of these formulas can be seen in the Appendix.

There is considerable controversy among academics and researchers about the right discount rate for the debt tax shields, the focus is its risk; if you assume that the debt tax shield has the same risk that the debt, then the appropriate rate of discount is Kd; if you assume that the debt tax shield bears the operational risk, then the appropriate rate is Ku. In their seminal contribution, Modigliani & Miller (1963) assumed a riskless and perpetual debt, which was discounted at the risk free rate. Myers (1974) in introducing the Adjusted Present Value concept assumed that the appropriate rate is Kd. Miles & Ezzell (1980) argue that for a firm wishing to keep a fixed leverage target, the appropriate discount rate for the debt tax shield is Kd for the first year (because the debt for the first year was known) and Ku for the following years. Harris & Pringle (1985) argued that Ku was an appropriate middle ground for the propositions of Modigliani & Miller (1963) and Miller (1977), which neglected the value of the debt tax shields. Inselbag & Kaufold (1997) claim that if the firm targets the dollar value of outstanding debt, Myers (1974) assumption was correct but if the firm targets a constant leverage, Miles & Ezzell (1980) assumptions were correct. Tham & Vélez-Pareja (2001), following and arbitrage argument, find that the appropriate rate of discount is Ku. Fernandez (2004), controversially, argue that the debt tax shield should be calculated as the unlevered cost of equity times debt times the tax rate times ($Ku \times D \times T$) and that the appropriate discount rate is Ku.

Our derivations do not depend on any of the particular assumptions we just described. Instead, we solve our set of equations for the different assumptions that are standard in the literature. Because our innovation is the equity tax shield, we also use those assumptions to produce the explicit expressions of our model. In section 3 we show that our assumptions produce consistent valuation numbers, independent of the method used for the calculations.

The Cost of Levered Equity

The general expression for Ke is

$$\begin{aligned} Ke_t &= Ku_t + (Ku_t - Kd_t)\frac{D_{t-1}}{E_{t-1}} - \left(Ku_t - \psi_t^D\right)\frac{V_{t-1}^{TSD}}{E_{t-1}} \\ &- \left(Ku_t - \psi_t^E\right)\frac{V_{t.1}^{TSE}}{E_{t-1}} \end{aligned} \quad (4)$$

If $\psi^D = \psi^E = Ku$ then

$$Ke_t = Ku_t + (Ku_t - Kd_t)\frac{D_{t-1}}{E_{t-1}} \quad (5a)$$

If $\psi^D = \psi^E = Ku$ then

$$Ke_t = Ku_t + (Ku_t - Kd_t)[\frac{D_{t-1} - V_{t-1}^{TSD} - V_{t-1}^{TSE}}{E_{t-1}}] \quad (5b)$$

If $\psi^D = Kd$ and $\psi^E = Ke$ then

$$Ke_t = Ku_t + (Ku_t - Kd_t)[\frac{D_{t-1} - V_{t-1}^{TSD} - V_{t-1}^{TSE}}{E_{t-1}}] + (Ku_t - Ke_t)[\frac{V_{t-1}^{TSE}}{E_{t-1}}] \tag{5c}$$

Observe that we can obtain a constant Ke only when leverage is constant and $\psi^D = \psi^E = Ku$.

In summary:

Table 2
Different formulations for Ke, the cost of levered equity

Formula	CFE
General	$Ke_t = Ku_t + (Ku_t - Kd_t)\frac{D_{t-1}}{E_{t-1}}$ $-(Ku_t - \psi_t^D)\frac{V_{t-1}^{TSD}}{E_{t-1}} - (Ku_t - Psi_t^E)\frac{V_{t-1}^{TSE}}{E_{t-1}}$
$\psi^D = \psi^E = Kd$	$Ke_t = Ku_t + (Ku_t - Kd_t)\left[\frac{D_{t-1} - V_{t-1}^{TSD} - V_{t-1}^{TSE}}{E_{t-1}}\right]$
$\psi^D = Kd$	$Ke_t = Ku_t + (Ku_t - Kd_t)\left[\frac{D_{t-1} - V_{t-1}^{TSD}}{E_{t-1}}\right]$ $+(Ku_t - Ke_t)\left[\frac{V_{t-1}^{TSE}}{E_{t-1}}\right]$

b. The General Formulation for Weighted Average Cost of Capital for the FCF

The general expression for $WACC_{gen}$ for the FCF is

$$WACC_{gent}^{FCF} = Ku_t - \left(Ku_t - \psi_t^D\right) \times \frac{V_{t-1}^{TSE}}{V_{t-1}^1} - \frac{TS_t^D + TS_t^E}{V_{t-1}^1} \tag{6}$$

If $\psi^D = \psi^E = Ku$ then

$$WACC_{gent}^{FCF} = Ku_t - \frac{TS_t^D + TS_t^E}{V_{t-1}^1} \tag{7a}$$

If $\psi^D = \psi^E = Kd$ then

$$WACC_{gent}^{FCF} = Ku_t - (Ku_t - Kd_t) \times \left[\frac{V_{t-1}^{TSD} + V_{t-1}^{TSE}}{V_{t-1}^1}\right] - \frac{TS_t^D + TS_t^E}{V_{t-1}^1} \tag{7b}$$

If $\psi^D = Kd$ and $\psi^E = Ke$ then

$$WACC_{gent}^{FCF} = Ku_t - (Ku_t - Kd_t) \times \frac{V_{t-1}^{TSD}}{V_{t-1}^L} - (Ku_t - Ke_t) \times \frac{TS_t^D + TS_t^E}{V_{t-1}^L} \tag{7c}$$

Observe that with this assumption we cannot obtain a constant $WACC$ when leverage is constant.

In summary:

Table 3
Different Formulations for WACC for the FCF

Formula	FCF
General	$WACC^{FCF}_{gent} = Ku_t - (Ku_t - \psi^D_t) \times \frac{V^{TSE}_{t-1}}{V^L_{t-1}} - \frac{TS^D_t + TS^E_t}{V^L_{t-1}}$
$\psi^D = \psi^E = Ku$	$WACC^{FCF}_{gent} = Ku_t - \frac{TS^D_t}{V^L_{t-1}} - \frac{TS^E_t}{V^L_{t-1}}$
$\psi^D = \psi^E = Kd$	$WACC^{FCF}_{gent} = Ku_t - (Ku_t - Kd_t) \times \left[\frac{V^{TSD}_{t-1} + V^{TSE}_{t-1}}{V^L_{t-1}}\right] - \frac{TS^D_t + TS^E_t}{V^L_{t-1}}$
$\psi^D = Kd$ and $\psi^E = Ke$	$WACC^{FCF}_{gent} = Ku_t - (Ku_t - Kd_t) \times (Ku_t - Ke_t) \times \frac{V^{TSE}_{t-1}}{V^L_{t-1}} - \frac{TS^D_t + TS^E_t}{V^L_{t-1}}$

c. The General Formulation for Weighted Average Cost of Capital for the CCF

The general expression for $WACC_{gen}$ for the CCF is

$$WACC^{FCF}_{gent} = Ku_t - \left(Ku_t - \psi^D_t\right) \times \frac{V^{TSD}_{t-1}}{V^L_{t-1}} - \left(Ku_t - \psi^E_t\right) \times \frac{V^{TSE}_{t-1}}{V^L_{t-1}} \quad (8)$$

If $\psi^D = \psi^E = Ku$ then

$$WACC^{CCF}_{gent} = Ku_t \quad (9a)$$

If $\psi^D = \psi^E = Kd$ then

$$WACC^{CCF}_{gent} = Ku_t - (Ku_t - Kd_t) \times \frac{V^{TSD}_{t-1} + V^{TSE}_{t-1}}{V^L_{t-1}} \quad (9b)$$

If $\psi^D = Kd$ and $\psi^E = Ke$ then

$$WACC^{CCF}_{gent} = Ku_t - (Ku_t - Kd_t) \times \frac{V^{TSD}_{t-1}}{V^L_{t-1}} - (Ku_t - Ke_t) \times \frac{V^{TSE}_{t-1}}{V^L_{t-1}} \quad (9c)$$

In summary

Table 4
Different formulations for WACC for the CCF

Formula	CCF
General	$WACC^{CCF}_{gent} = Ku_t - (Ku_t - \psi^D_t) \times \frac{V^{TSD}_{t-1}}{V^L_{t-1}} - (Ku_t - \psi^E_t) \times \frac{V^{TSE}_{t-1}}{V^L_{t-1}}$
$\psi^D = \psi^E = Ku$	$WACC^{CCF}_{gent} = Ku_t$
$\psi^D = \psi^E = Kd$	$WACC^{CCF}_{gent} = Ku_t - (Ku_t - Kd_t) \times \left[\frac{V^{TSD}_{t-1}+V^{TSE}_{t-1}}{V^L_{t-1}}\right]$
$\psi^D = Kd$ and $\psi^E = Ke$	$WACC^{CCF}_{gent} = Ku_t - (Ku_t - Kd_t) \times (Ku_t - Kd_t) \times \frac{V^{TSD}_{t-1}}{V^L_{t-1}} - (Ku_t - Ke_t) \times \frac{V^{TSE}_{t-1}}{V^L_{t-1}}$

The derivation of these formulas can be seen in the Appendix.

Observe that we can obtain a constant $WACC$ for the CCF, $WACC_{CCF}$, only when leverage is constant and $\psi^D = \psi^E = Ku$.

3. A Finite Cash Flows Example

Next we show a finite cash flow example. In this example we consider four methods to calculate value: DCF with Ke for the CFE; DCF with $WACC$ for the FCF; DCF with the $WACC$ for the CCF; and Adjusted Present Value, APV for three scenarios for the discount rate of tax shields.

Table 5
Input data 1 – basic inputs: beta and rates

T	40%	βu	1
Kd	12%	Rf	7%
Interest on equity’	8%	Rm-Rf	7%
		Ku	14.0%

Another input data is related to the cash flows and debt balances.

Table 6
Input data 2 – cash flows

	0	1	2	3	4	5
Debt, D	100.00	80.00	60.00	40.00	20.00	–
Debt Payment		20.00	20.00	20.00	20.00	20.00
Interest on D		12.00	9.60	7.20	4.80	2.40
CFD		32.00	29.60	27.20	24.80	22.40
TS^D		4.80	3.84	2.88	1.92	0.96
FCF		40.00	42.00	44.10	46.31	48.62
BVE	100.00	100.00	100.00	100.00	100.00	100.00
Interest on BVE		8.00	8.00	8.00	8.00	8.00
TSE		3.20	3.20	3.20	3.20	3.20
CCF		48.00	49.04	50.18	51.43	52.78
CFE		16.00	19.44	22.98	26.63	30.38

With this information we calculate the firm value for different scenarios of the discount rate for TS.

Table 7
Value calculations 1 – discount rate for $TS = Ku$

If $\psi^D = \psi^E = Ku$						
	0	1	2	3	4	5
Ke		16.79%	16.37%	16.03%	15.75%	15.52%
E	71.57	67.59	59.21	45.72	26.30	
$V = D + E$	**171.57**	147.59	119.21	85.72	46.30	
$WACC^{FCF}$		9.34%	9.23%	8.90%	8.03%	5.01%
V	**171.57**	147.59	119.21	85.72	46.30	
D	100.000	80.000	60.000	40.000	20.000	–
$WACC^{CCF} = Ku$		14.0%	14.0%	14.0%	14.0%	14.0%
$PVCCF$	**171.57**	147.59	119.21	85.72	46.30	
V^{Un}	149.84	130.82	107.13	78.03	42.65	
V^{TSD}	10.74	7.45	4.65	2.42	0.84	
V^{TSE}	10.99	9.32	7.43	5.27	2.81	
APV	**171.57**	147.59	119.21	85.72	46.30	

Table 8
Value calculations 2 – discount tate for $TS = Kd$

If $\psi^D = \psi^E = Kd$	0	1	2	3	4	5
V^{TSD}	11.16	7.70	4.79	2.48	0.86	
V^{TSE}	11.54	9.72	7.69	5.41	2.86	
Ke		16.13%	15.83%	15.59%	15.40%	15.24%
E	72.54	68.24	59.60	45.92	26.36	
$V = D + E$	**172.54**	148.24	119.60	85.92	46.36	
$WACC^{FCF}$		9.10%	9.02%	8.71%	7.86%	4.87%
V	**172.54**	148.24	119.60	85.92	46.36	
D	100.000	80.000	60.000	40.000	20.000	
$WACC^{CCF}$		13.74%	13.76%	13.79%	13.82%	13.84%
$PVCCF$	**172.54**	148.24	119.60	85.92	46.36	
V^{Un}	149.84	130.82	107.13	78.03	42.65	
V^{TSD}	11.16	7.70	4.79	2.48	0.86	
V^{TSE}	11.54	9.72	7.69	5.41	2.86	
APV	**172.54**	148.24	119.60	85.92	46.36	

Table 9
Value calculations 3 – discount rate for $TS = Kd$ and Ke

If $\psi^D = \psi^E = Kd$	0	1	2	3	4	5
V^{TSD}	11.16	7.70	4.79	2.48	0.86	
V^{TSE}	10.37	8.92	7.19	5.15	2.77	
Ke		16.91%	16.47%	16.13%	15.85%	15.63%
E	71.37	67.44	59.11	45.66	26.27	
$V = D + E$	**171.37**	147.44	119.11	85.66	46.27	
$WACC^{FCF}$		9.38%	9.27%	8.94%	8.08%	5.07%
V	**171.37**	147.44	119.11	85.66	46.27	
D	100.000	80.000	60.000	40.000	20.000	
$WACC^{CCF}$		14.05%	14.05%	14.05%	14.05%	14.06%
$PVCCF$	**171.37**	147.44	119.11	85.66	46.27	
V^{Un}	149.84	130.82	107.13	78.03	42.65	
V^{TSD}	11.16	7.70	4.79	2.48	0.86	
V^{TSE}	10.37	8.92	7.19	5.15	2.77	
APV	**171.37**	147.44	119.11	85.66	46.27	

As can be seen for each set of assumptions on discount rate for TS, the four methods give consistent and identical results as expected.

Although it is not the purpose of this paper, we can say something regarding the risk for the tax savings. In case of debt, we have two situations:

- Case 1. Leverage $D\%$ is defined as a target leverage and $Debt = D\% \times V_{t-1}$ and V depends on FCF; hence the risk of TS should be Ku.
- Case 2. Debt and the CFD profile are defined. Again, TS depends on FCF (or EBIT). In Vélez-Pareja (2010) it is shown that if $EBIT > Financial$

$Expenses(FE) TS = T \times FE$; if $0 < EBIT < GF, TS = T \times EBIT$; if $EBIT < 0, TS = 0$. Hence, TS depends on EBIT and its risk should be Ku, the cost of unlevered equity (that is the risk of FCF).

4. Empirical Predictions

Our paper produces at least two empirical predictions that we underline in this section. It has been established, long ago and in this paper, that higher debt produces higher tax shields from debt, as long as operating profit is greater than interest payments. We developed in this paper expressions for an additional tax shield, deductible dividends. However, the trade-off theory states that with higher debt comes an increasing probability of financial distress. Then, with deductible dividends the expression for the levered firm becomes:

$$V^L = V^U + PV(TS^D) + PV(TS^E) - BC \tag{10}$$

The levered firm value is the sum of the unlevered firm, the present value of the tax shields of debt and equity and the bankruptcy costs. We assume that bankruptcy costs increase with the level of debt at an increasing rate $\left(\frac{\delta BC}{\delta D}, \frac{\delta^2 BC}{\delta D^2} > 0\right)$, which means that the dangers of leverage speed up, as usual.

With constant and perpetual cash flows[7] equation 10 becomes:

$$V^L = V^U + \frac{T_d D K_d}{\psi^D} + \frac{T_e E K'_f}{\psi^E} - BC \tag{11}$$

The former expression,[8] besides already defined terms like Debt (D) and Equity (E) its respective costs ($K_d, K_f^{'}$) and the appropriate discount rates (ψ^D, ψ^E), includes T_d and T_e, which are modified tax rates when personal taxes are included:

$T_d = (1 - \tau_{PB}) - (1 - T)(1 - \tau_{PS})$ and $T_e = (1 - \tau_{PDiv}) - (1 - T)(1 - \tau_{PS})$

where τ_{PB}, τ_{PS} and τ_{PDiv} are the personal tax rate for holding debt, shares, and income for deductible dividends, respectively. T is the corporate tax rate. τ_{PS} comprises both capital gains taxes and dividends taxes.

We can solve equation 11 using two different assumptions. First, we can apply the Brazilian approach which works with book values: $E = A - D$. Factoring the last identity in equation 11 produces:

$$V^L = V^U + D\left(T_d \frac{K_d}{\psi^D} - T_e \frac{K_f^{'}}{\psi^E}\right) + \frac{T_e A K_f^{'}}{\psi^E} - BC \tag{12a}$$

[7] See the Chapter 15, Section B, of Copeland *et al.* (2005) for a further clarification of the equation.

[8] Equation 11 differs from the corresponding equation in Zani & Ness (2001), because we do not neglect τ_{PS} in the term T_e.

Equation 12a tells us that, while the tax shield from the debt decreases, a whole new term more than compensates for the reduction.

If we use market values: $E = V^L - D$, we obtain the following expression:

$$V^L = \frac{\left(V^U + D\left(T_d \frac{K_d}{\psi^D} - T_e \frac{K_f^{'}}{\psi^E}\right) - BC\right)}{1 - T_e \frac{K_f^{'}}{\psi^E}} \tag{12b}$$

Equations 12b show us the double effect of deductible dividends, while the tax shield value is reduced, it does not disappear while $T_d \frac{K_d}{\psi^D} > T_e \frac{K_f^{'}}{\psi^E}$. In the case of Brazil, the Brazilian Central Bank sets the high limit of $K_f^{'}$ as below the long-term interest rate[9] (TJLP), which, with almost certainty, is below $K_d \leq \psi^D \leq \psi^E$ the cost of debt, the discount rate for debt interest and tax savings from equity. The condition for $T_d > T_e$ is that $\tau_{PDiv} > \tau_{PB}$, which is plausible given the risk profiles of each investment.

Additionally, the firm value increases in $\frac{1}{1 - T_e \frac{K_f^{'}}{\psi^E}}$. We also hypothesize that in most conditions the latter effect dominates the first one.

No matter what assumption regarding the equity value we use, the partial derivative of equation 11 gives us the conditions for the optimal level of debt, assuming that all discount rates are exogenous:

$$\frac{\delta BC}{\delta D} = T_d \frac{K_d}{\psi^D} - T_e \frac{K_f'}{\psi^E} \tag{13}$$

In absence of an equity tax shield $(K_f^{'} = 0)$, and if the conditions previously analyzed stand, the optimal level of debt is higher.[10]

The conclusions of this section are that under plausible conditions the firm value increases and that the optimal level of debt is lower in a legal regime that allows firms to deduct part of their dividends from their taxes. Zani & Ness (2001) reach a similar conclusion using a numerical example; our results generalize their argument and can be extended for additional tax shields. As stated in the introduction, Damodaran (2003) discusses a similar issue in a working paper that analyzes the proposed Economic Package of the Bush administration. The package was approved by the American Senate in 2003 under significant changes. Before 2003 dividends were fully taxable at the ordinary rate, after the amendment the tax on dividends was reduced to 15% until 2010, but firms were not allowed to subtract

[9]According to the Brazilian rules, the rate corporations can declare should be below TJLP, which is calculated quarterly as the expected inflation plus a risk spread on the country's sovereign debt. This rate has remained unchanged since the third quarter of 2009 in 6%.

[10]For Zani & Ness (2001) the indifference condition is $1 - \tau_{PB} = (1 - T)(1 - \tau_{PS}) + (T - \tau_{PDiv})/2$, ours is $T_d \frac{K_d}{\psi^D} = T_e \frac{K_f^{'}}{\psi^E}$. Zani & Ness (2001) assume that Ku is the right discount rate for the deductible dividends, we do not.

paid dividends as a taxable expense. The modifications to the original Modigliani & Miller (1963) model we study here can be classified as a formalization of the efforts governments do to promote the use of equity over debt, which the business community witness from time to time (Damodaran, 2003, Bernasconi *et al.*, 2005). This paper emphasizes the consequences this kind of policies has on the valuation of enterprises.

5. Concluding Remarks

This paper analyzed the formulation of $WACC$ and Ke under scenarios with tax savings originated by items different than the interest charges on debt. We have derived the formulations in a general way and they can be used for finite and perpetuities cash flows. For the later, we have to recognize the value of TS in a given perpetuity scenario. We show an example for finite cash flows. In this example we show that four methods give consistent results: i) Firm value with Free Cash Flow, FCF and $WACC$ for the FCF, $WACC_{FCF}$; ii) value with the Capital Cash Flow, CCF; iii) equity value with the Cash Flow to Equity and Ke, the levered cost of equity plus debt; iv) Adjusted Present Value, APV.

We calculated value for three scenarios depending on the discount rate ψ for TS from two sources: interest charges on debt and interest charges on book value of equity. The value for ψ was Kd and Ku for both tax savings and a third one that assumes Kd and Ke for TS from debt and equity respectively.

The formulations work for any debt profile: constant debt, variable debt or constant or target leverage. From the formulations as mentioned in Vélez-Pareja *et al.* (2008), constant leverage does not grant that $WACC$ or Ke be constant. It depends on how the TS affect the respective formulation. The only formulas whose value remains constant with constant leverage are Ke and $WACC_{CCF}$ when the discount rate ψ, for both tax savings is equal to Ku. We also analyze which should be the proper discount rate for the two tax savings (debt and book value of equity). We suggest that it should be the cost of unlevered equity, Ku.

Finally, we solve for the optimal leverage in a model that assumes that the Trade-off theory holds. Under this scenario firms are less leveraged, granting a relevant empirical question that we leave for further research: We might examine the effect of tax shields earned by equity (interest on equity or adjustment of equity when financial statements are adjusted by inflation) on the capital structure in different countries including Brazil with the two scenarios: adjustment of equity (when Brazil used to adjust financial statements by inflation) and interest on equity (as a part of dividends paid).

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Appendix

Derivation of Traditional Textbook formula for WACC for the FCF

The discount rate for free cash flows is the weighted average cost of capital $WACC$. We derive its formulation using a modified version of the equilibrium equation for values. Firm value is (Exhibit 1) the value of operations, Vu plus the value of TS from debt (V^{TSD}) plus the value of TS for equity (V^{TSE}). This value should be identical to the value of debt holders plus equity. Each element is associated to a discount rate according to its risk profile.

Exhibit 1. The firm in terms of assets and fund providers

V^{Un} (Ku)	D (Kd)
V^{TSD} (ψ^D)	
V^{TSE} (ψ^E)	E (Ke)

The Cost of Equity, Ke

The treatment for perpetuities and finite cash flows is basically the same. When we deal with finite cash flows we relate a value at t with its value at $t+1$ multiplying the value at t by (1+discount rate). However, when we realize that $Vu + VTSd + VTSe = D + E$, the 1 inside the parenthesis multiplies exactly these values at the left and the right hand sides of the equation and they cancel each other. The net result is that the formulation is the same for finite cash flows and for perpetuities. Care has to be taken when calculating the values involved in the formulation. In particular, when we are calculating the value of the tax shields, for instance, the value of tax savings for interest expenses when we deal a perpetuity and the discount rate for the TS is Kd. The TS is $T \times D_{t-1} \times Kd_t$. If we deal with a perpetuity, the present value of a perpetuity of $T \times D_{t-1} \times Kd_t$ is $T \times D_{t-1} \times Kd_t / Kd_t = T \times D_{t-1}$. When dealing with finite cash flows we do not know the value in a standard from as in the perpetuity and has to be calculated depending on the future TS within the planning horizon.

$$FCF_t + TS_t^D + TS_t^E = CFD_t + CFE_t \tag{A.1}$$

$$Vu_t + V_t^{TSD} + V_t^{TSE} = D_t + E_t \tag{A.2}$$

$$Vu_t = D_t + E_t - V_t^{TSD} - V_t^T SE \tag{A.3}$$

$$Vu_{t-1} \times Ku_t + V_{t-1}^{TSD} \times D + V_{t-1}^{TSE} \times E = D_{t-1} \times Kd_t + E_{t-1} \times Ke_t \tag{A.4a}$$

Solving for Ke_t

$$E_{t-1} \times Ke_t = Vu_{t-1} \times Ku_t + V_{t-1}^{TSD} \times D + V_{t-1}^{TSE} \times E - D_{t-1} \times Kd_t \quad \text{(A.4b)}$$

Replacing Vu_{t-1} by it value

$$\begin{aligned} E_{t-1} \times Ke_t = & \quad (D_{t-1} + E_{t-1} - V_{t-1}^{TSD} - V_{t-1}^{TSE})_t \times Ku_t \\ + & \quad V_{t-1}^{TSD} \times D + V_{t-1}^{TSE} \times E - D_{t-1} \times Kd_t \end{aligned} \quad \text{(A.4c)}$$

Dividing by E_{t-1}

$$\begin{aligned} Ke_t = & \quad (\frac{D_{t-1}}{E_{t-1}} + 1 - \frac{V_{t-1}^{TSD}}{E_{t-1}} - \frac{V_{t-1}^{TSE}}{E_{t-1}}) \times Ku_t \\ + & \quad \frac{V_{t-1}^{TSD} \times \psi^D}{E_{t-1}} + \frac{D_{t-1} \times Kd_t}{E_{t-1}} \end{aligned} \quad \text{(A.4d)}$$

Simplifying

$$\begin{aligned} Ke_t = Ku_t + (\frac{D_{t-1}}{E_{t-1}} - \frac{V_{t-1}^{TSD}}{E_{t-1}} - \frac{V_{t-1}^{TSE}}{E_{t-1}}) \times Ku_t \\ + \frac{V_{t-1}^{TSD} \times \psi^D}{E_{t-1}} + \frac{V_{t-1}^{TSE} \times \psi^E}{E_{t-1}} - \frac{D_{t-1} \times Kd_t}{E_{t-1}} \end{aligned} \quad \text{(A.4e)}$$

Grouping we find the general expression for Ke

$$\begin{aligned} Ke_t = & \quad Ku_t + (Ku_t - Kd_t) \times \frac{D_{t-1}}{E_{t-1}} \\ - & \quad (Ku_t - \psi^D) \times \frac{V_{t-1}^{TSD}}{E_{t-1}} - (Ku_t - \psi^E) \times \frac{V_{t-1}^{TSE}}{E_{t-1}} \end{aligned} \quad \text{(A.4f)}$$

We present selected values for the discount rate of the TS.

- If $\psi^D = \psi^E = Ku$

$$Ke_t = Ku_t + (Ku_t - Kd_t) \times \frac{D_{t-1}}{E_{t-1}} \quad \text{(A.5)}$$

- If $\psi^D = \psi^E = Kd$

$$Ke_t = Ku_t + (Ku_t - Kd_t) \times \frac{D_{t-1}}{E_{t-1}} - (Ku_t - Kd) \times \frac{V^{TSD}}{E_{t-1}} \quad \text{(A.6)}$$

$$Ke_t = Ku_t + (Ku_t - Kd_t) \times \left(\frac{D_{t-1}}{E_{t-1}} - \frac{V^{TSD}_{t-1}}{E_{t-1}} - \frac{V^{TSE}_{t-1}}{E_{t-1}} \right) \quad \text{(A.7)}$$

- As $V^{TSD} = TD_{t-1}$ in perpetuity when $\psi^D = \psi^E = Kd$

$$Ke_t = Ku_t + (Ku_t - Kd_t) \times (\frac{D_{t-1}}{E_{t-1}} - \frac{TXD_{t-1}}{E_{t-1}} - \frac{V^{TSE}_{t-1}}{E_{t-1}} \quad \text{(A.8a)}$$

$$Ke_t = Ku_t + (Ku_t - Kd_t) \times (1 - T) \times \frac{D_{t-1}}{E_{t-1}} - (Ku_t - Kd_t) \times \frac{V^{TSE}_{t-1}}{E_{t-1}} \quad \text{(A.8b)}$$

- If $\psi^D = \psi^E = Kd = Kf$ and the interest rate for equity is Kf, the risk free rate, and $E_i = EBV$ (book value of E) and EBV is the basis for calculating interest on equity then

$$Ke_t = Ku_t + (Ku_t - Kf_t) \times \frac{TXD_{t-1}}{E_{t-1}} - (Ku_t - Kf_t) \times \frac{TXBVE_{t-1}}{E_{t-1}} \quad \text{(A.9a)}$$

$$Ke_t = Ku_t + (Ku_t - Kf_t) \times (\frac{D_{t-1}}{E_{t-1}} - \frac{T \times D_{t-1}}{E_{t-1}} - \frac{T \times BVE_{t-1}}{E_{t-1}}) \quad \text{(A.9b)}$$

- If Ei, the basis for calculating interest on equity is $Ei = E$ (E = market value of equity)

$$Ke_t = Ku_t + (Ku_t - Kf_t) \times (\frac{D_{t-1}}{E_{t-1}} - \frac{T \times D_{t-1}}{E_{t-1}} - T) \quad \text{(A.10)}$$

- If $\psi^D = Kd$ and $\psi^E = Ke$ then

$$\begin{aligned} Ke_t = Ku_t + (Ku_t - Kd_t)\frac{D_{t-1}}{E_{t-1}} \\ -(Ku_t - Kd)\frac{V^{TSD}}{E_{t-1}} - (Ku_t - Ke)\frac{V^{TSE}_{t-1}}{E_{t-1}} \end{aligned} \quad \text{(A.11a)}$$

- Solving for Ke

$$Ke_t - Ke \times \frac{V_{t.1}^{TSE}}{E_{t-1}} = Ku_t + (Ku_t - Kd_t)\frac{D_{t-1}}{E_{t-1}}$$
$$-(Ku_t - Kd)\frac{V_{t-1}^{TSD}}{E_{t-1}} - Ku_t \times \frac{V_{t-1}^{TSE}}{E_{t-1}} \qquad \text{(A.11b)}$$

Grouping

$$Ke_t \times (1 - \frac{V_{t-1}^{TSE}}{E_{t-1}} = Ku_t + (Ku_t - Kd_t)\frac{D_{t-1}}{E_{t-1}}$$
$$-(Ku_t - Kd_t)\frac{V_{t-1}^{TSD}}{E_{t-1}} - Ku_t \times \frac{V_{t-1}^{TSE}}{E_{t-1}} \qquad \text{(A.11c)}$$

Dividing by $\left(1 - \frac{V_{t-1}^{TSE}}{E_{t-1}}\right)$ and simplifying

$$Ke_t = \frac{E_{t-1} \times Ku_t}{E_{t-1} - V_{t-1}^{TSE}} + (Ku_t - Kd_t)\frac{D_{t-1}}{E_{t-1} - V_{t-1}^{TSE}}$$
$$-(Ku_t - Kd)\frac{V_{t-1}^{TSD}}{E_{t-1} - V_{t-1}^{TSE}} - Ku_t \times \frac{V_{t-1}^{TSE}}{E_{t-1} - V_{t-1}^{TSE}} \qquad \text{(A.12a)}$$

Grouping and simplifying

$$Ke_t = Ku \times \frac{E_{t-1}}{E_{t-1} - V_{t-1}^{TSE}} - Ku_t \times \frac{V_{t-1}^{TSE}}{E_{t-1} - V_{t-1}^{TSE}}$$
$$+(Ku_t - Kd_t)\frac{D_{t-1}}{E_{t-1} - V_{t-1}^{TSE}} - (Ku_t - Kd)\frac{V_{t-1}^{TSD}}{E_{t-1} - V_{t-1}^{TSE}} \qquad \text{(A.12b)}$$

$$Ke_t = Ku_t + (Ku_t - Kd_t)\left[\frac{D_{t-1} - V_{t-1}^{TSD}}{E_{t-1} - V_{t-1}^{TSE}}\right] \qquad \text{(A.12c)}$$

Traditional Textbook Formula for WACC for the FCF

The textbook formula has many restrictions and assumptions, for instance:

1 The only source of tax savings (TS) is interest on debt.

2 Taxes are paid the same period when accrued.

3 Existence of enough EBIT + Other income to earn the TS.

For the textbook formula for WACC the cash flows from the left hand side (assets) should be identical to the cash flows at the right hand side.

We depart from the basic M&M relations among CFs

$$FCF_t + TS_t^D + TS_t^E = CFD_t + CFE_t \tag{A.13}$$

$$FCF_t = CFD_t + CFE_t - TS_t^D - TS_t^E \tag{A.14}$$

For the FCF and WACC

$$V_t \times WACC_{t+1} = Kd_{t+1} \times D_t + Ke_{t+1} \times E_t - Kd_{t+1} \times D_t \times T - Kf_{t+1} \times E_t \times T \tag{A.15a}$$

$$WACC_{t+1} = Kd_{t+1} \times (1-T) \times D\%_t + Ke_{t+1} \times E\%_t - Kf_{t+1} \times (1-T) \times E\%_t \tag{A.15b}$$

The value of the firm is increased by the TS on interest on equity.

General WACC applied to the FCF

Formulating WACC in a general formulation eliminates some restrictions associated to the traditional textbook WACC, as mentioned above.

Let $WACC_i^{Gen}$ be the General WACC that is applied to the FCF in year i. We follow the same steps that we outlined for the standard WACC applied to the FCF and obtain an equation that is similar to equation 22.$\times$

$$V_{t-1}^L \times WACC_t^{Gen} = D_{t-1} \times Kd_t - TS_t + E_{t-1}^L \times Ke_t \tag{A.16a}$$

$$V_{t-1}^L \times WACC_t^{Gen} = V_{t-1}^{Un} \times Ku_t + V_{t-1}^{TS} \times \psi_t - TS_t \tag{A.16b}$$

$$V_{t-1}^L \times WACC_t^{Gen} = (V_{t-1}^L - V_{t-1}^{TS}) \times Ku_t + V_{t-1}^{TS} \times \psi_t - TS_t \tag{A.16c}$$

$$V_{t-1}^L \times WACC_t^{Gen} = V_{t-1}^L \times Ku_t - (Ku_t - \psi_t) \times V_{t-1}^{TS} - TS_t \tag{A.16d}$$

Solving for the WACC in equation (A.16d), we obtain,

$$WACC_t^{Gen} = Ku_t - (Ku_t - \psi_t) \times \frac{V_{t-1}^{TS}}{V_{t-1}^L} - \frac{TS_t}{V_{t-1}^L} \tag{A.16f}$$

If we assume that the value of ψ_i is equal to the return to unlevered equity Ku_i, we can simplify equation (A.16f) as follows.

$$WACC_t^{Gen} = Ku_t - \frac{TS_t}{V_{t-1}^L} \tag{A.17}$$

Alternatively, if we assume that the value of ψ_i is equal to the cost of debt di, we can write equation (A.16f) as follows.

$$WACC_t^{Gen} = Ku_t - (Ku_t - Kd_t) \times \frac{V_{t-1}^{TS}}{V_{t-1}^L} - \frac{TS_t}{V_{t-1}^L} \tag{A.18}$$

This previous derivation is taken from Tham & Vélez-Pareja (2004).

We might think that ALL TS have the same discount rate, which is not too "elegant". In that case we apply the previous formulation. The best and general approach is to work with a general formulation of WACC instead of the traditional textbook formulation that is specific for a particular case.

Repeating the procedure shown above, we have:

We depart from the basic equilibrium equations for cash flows and values:

$$FCF + TS = CFD + CFE \tag{A.19}$$

$$V^{Un} + V^{TS} = D + E \tag{A20}$$

$$V_{t-1}^L \times WACC_t^{Gen} = D_{t-1} \times Kd_t - TS_t^D - TS_t^E + E_{t-1}^L \times Ke_t \tag{A.21a}$$

Replacing the cash flows for D and E by the corresponding LHS cash flows we have:

$$\begin{aligned} V_{t-1}^L \times WACC_t^{Gen} = {} & V_{t-1}^{Un} \times Ku_t + V_{t-1}^{TSD} \times \psi_t^D \\ & + V_{t-1}^{TSE} \times \psi_t^E - TS_t^D - TS_t^E \end{aligned} \tag{A.21b}$$

Replacing the unlevered value by the levered value minus the value of TS, we have

$$\begin{aligned} V_{t-1}^L \times WACC_{gen} = {} & (V_{t-1}^L - V_{t-1}^{TSD} - V_{t-1}^{TSE}) \times Ku_t \\ & + V_{t-1}^{TSD} \times \psi_t^D + V_{t-1}^{TSE} \times \psi_t^E - TS_t^D - TS_t^E \end{aligned} \tag{A.21c}$$

Solving for WACC we obtain,

$$WACC_t^{Gen} = (\frac{V_{t-1}^L}{V_{t-1}^L} - \frac{V_{t-1}^{TSD}}{V_{t-1}^L} - \frac{V_{t-1}^{TSE}}{V_{t-1}^L}) \times Ku_t$$
$$+\frac{V_{t-1}^{TSD}}{V_{t-1}^L} \times \psi_t^D + \frac{V_{t-1}^{TSE}}{V_{t-1}^L} \times \psi_t^E - \frac{TS_t^D}{V_{t-1}^L} - \frac{TS_t^E}{V_{t-1}^L} \quad \text{(A.21d)}$$

Developing the term inside parenthesis, multiplying by Ku and grouping we have

$$WACC_t^{Gen} = Ku_t - Ku_t \times \frac{V_{t-1}^{TSD}}{V_{t-1}^L} - Ku_t \times \frac{V_{t-1}^{TSE}}{V_{t-1}^L} \times Ku_t$$
$$+\frac{V_{t-1}^{TSD}}{V_{t-1}^L} \times \psi_t^D + \frac{V_{t-1}^{TSE}}{V_{t-1}^L} \times \psi_t^E - \frac{TS_t^D}{V_{t-1}^L} - \frac{TS_t^E}{V_{t-1}^L} \quad \text{(A.21e)}$$

$$WACC_t^{Gen} = Ku_t - (Ku_t - \psi_t^D) \times \frac{V_{t-1}^{TSD}}{V_{t-1}^L}$$
$$-(Ku_t - \psi_t^E) \times \frac{V_{t-1}^{TSE}}{V_{t-1}^L} - \frac{TS_t^D}{V_{t-1}^L} - \frac{TS_t^E}{V_{t-1}^L} \quad \text{(A.21f)}$$

This is the general formulation for $WACC$ for the FCF. Observe it has the same structure than the one developed above.

Now we define scenarios for the discount rate for the TS

- If $\psi_D = \psi^E = Ku$ then

$$WACC_t^{Gen} = Ku_t - \frac{TS_t^D}{V_{t-1}^L} - \frac{TS_t^E}{V_{t-1}^L} \quad \text{(A.22)}$$

- If $\psi^D = \psi^E = Kd$ then

$$WACC_t^{Gen} = Ku_t - (Ku_t - Kd_t) \times [\frac{V_{t-1}^{TSD}}{V_{t-1}^L} + \frac{V_{t-1}^{TSE}}{V_{t-1}^L}] - \frac{TS_t^D}{V_{t-1}^L} - \frac{TS_t^E}{V_{t-1}^L} \quad \text{(A.23)}$$

- $\psi^D = Kd$ and $\psi^D = Ke$ then

$$WACC_t^{Gen} = Ku_t - (Ku_t - Kd_t) \times \frac{V_{t-1}^{TSD}}{V_{t-1}^L}$$
$$-(Ku_t - Ke_t) \times \frac{V_{t-1}^{TSE}}{V_{t-1}^L} - \frac{TS_t^D}{V_{t-1}^L} - \frac{TS_t^E}{V_{t-1}^L} \quad \text{(A.24)}$$

General WACC applied to the CCF

We know that the CCF is equal to the sum of the FCF and the TS.

$$CCF_i = FCF_i + TS_i \tag{A.25}$$

Let $WACC_i^{Gen}$ be the general $WACC$ applied to the CCF. We follow the same steps that we outlined for the standard $WACC$ applied to the FCF.

$$V_{t-1}^{L} \times WACC_t^{Gen} = V_{t-1}^{Un} \times Ku_t + V_{t-1}^{TS} \times \psi_t \tag{A.26a}$$

$$V_{t-1}^{L} \times WACC_t^{Gen} = V_{t-1}^{L} \times Ku_t - (Ku_t - \psi_t) \times V_{t-1}^{TS} \tag{A.26b}$$

Solving for the WACC, we obtain,

$$WACC_t^{Gen} = Ku_t - (Ku_t - \psi_t) \times \frac{V_{t-1}^{TS}}{V_{t-1}^{L}} \tag{A.26c}$$

If we assume that the value of i is equal to the return to unlevered equity Kui, we can simplify equation (A.26c) as follows.

$$WACC_t^{Gen} = Ku_t \tag{A.27}$$

If we assume that the value of ψ_i is equal to the cost of debt Kd_i, we can write equation (A.26c) as follows.

$$WACC_t^{Gen} = Ku_t - (Ku_t - Kd_t) \times \frac{V_{t-1}^{TS}}{V_{t-1}^{L}} \tag{A.28}$$

This previous derivation is taken from Tham & Vélez-Pareja (2004).

In the same vein, if we asume that all TS (interest on equity and interest on debt) have the same risk (the same discount rate) we use the previous formulation.

When we introduce the two different sources of TS with their specific risks, we have

$$WACC_t^{Gen} = Ku_t - (Ku_t - \psi_t^{D}) \times \frac{V_{t-1}^{TSD}}{V_{t-1}^{L}} - (Ku_t - \psi_t^{E}) \times \frac{V_{t-1}^{TSE}}{V_{t-1}^{L}} \tag{A.29}$$

For different values of the discount rate for the TS

- If $\psi_D = \psi^E = Ku$ then

$$WACC_t^{Gen} = Ku_t \tag{A.30}$$

- If $\psi_D = \psi^E = Kd$ then

$$WACC_t^{Gen} = Ku_t - (Ku_t - Kd_t) \times [\frac{V_{t-1}^{TSD}}{V_{t-1}^L} - \frac{V_{t-1}^{TSE}}{V_{t-1}^L}] \qquad \text{(A.31)}$$

- If $\psi^D = Kd$ and $\psi^E = Ke$ then

$$\begin{aligned} WACC_t^{Gen} &= Ku_t - (Ku_t - Kd_t) \\ &\times \frac{V_{t-1}^{TSD}}{V_{t-1}^L} - (Ku_t - Ke_t) \times \frac{V_{t-1}^{TSE}}{V_{t-1}^L} \end{aligned} \qquad \text{(A.32)}$$