The UWB-OFDM Channel Analysis in Frequency

Gonzalo Llano, Juan Reig and Lorenzo Rubio
Departamento de Comunicaciones
Universidad Politécnica de Valencia
Camino de Vera s/n 46022 Valencia (Spain)
e-mail: gonllara@doctor.upv.es, jreigp@dcom.upv.es, lrubio@dcom.upv.es.

Abstract— In this paper, the ultra-wideband channel with orthogonal frequency division multiplexing (UWB-OFDM) is analyzed in the frequency domain. For UWB-OFDM channels with log-normal fading in the time domain, we show that the amplitude of each subcarrier can be approximated by a Nakagami-*m* random variable, where the fading parameter, the mean power and the correlation coefficient are expressed in terms of the following parameters: time arrival of the clusters, inter-arrival time of rays inside clusters, and power decay constants of rays and clusters.

Keywords-UBW-OFDM, log-normal, Nakagami-m fading

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is an important wideband and ultra-wideband transmission scheme for modern wireless communications systems. An OFDM system can reduce or eliminate intersymbol interference (ISI), and is particularly suitable for transmission over frequency selective fading channels requiring only a relatively simple equalizer at the receiver for a good performance [1]. On the other hand, diversity techniques (time, frequency, space, code) can improve the system performance, especially in the frequency-flat fading environment. The combination of ultra-wideband (UWB) with OFDM and diversity techniques has been becoming very important in wireless communications especially in indoor environments for high data rate wireless personal area networks (HDR-WPAN). Hence it is of interest to investigate the UWB-OFDM system performance in different fading channels [2].

Both Rayleigh and Rician fading models are often assumed in the system design and performance analysis in narrowband and wideband systems for modeling and characterizing the amplitude of fading channel [1]. More recently, the Nakagami-*m* distribution [3] has received considerable attention due to its good flexibility and accuracy in matching various experimental data more general than Raleigh, Rician or log-normal distributions [2]. Previous work that studied transmission of an OFDM signal over frequency selective Nakagami-*m* has assumed that the frequency domain channel response samples are also Nakagami-*m* distributed with same fading parameters as the time domain channel [4]. However, there are no experimental results presented in the literature that support this assumption [1]. Hence the study of the UWB channel in the frequency domain can be of great interest to analyze the performance of the UWB-OFDM system concerning to the channel estimation, and bit and symbol error analysis. Moreover, an accurate model in frequency of the UWB channel is required to design modulation adaptive techniques which increase the channel capacity and to achieve greater benefits and more efficient systems implementations.

The statistical description of the IEEE 802.15.3a UWB-OFDM channel for UWB communications systems in order to compare standardization proposal HDR-WPAN defines a modified Saleh-Valenzuela (S-V) [5] model to describe the arrival time of clusters and rays at the receiver after multipath propagation. In this model, the amplitude of clusters and rays follows a log-normal distribution [6], [7].

In this letter, we obtain a Nakagami-*m* approximate distribution of subcarriers envelope for the IEEE 802.15.3a UWB model. Additionally, the power correlation coefficient between a couple of subcarriers amplitudes is calculated analytically for this model. To the author's knowledge these results are novel in the literature of UWB channel modeling especially using the IEEE 802.15.3a model. These results could substantially improve the analysis and design of the MB-OFDM-UWB system.

II. UWB CHANNEL AND IEEE 802.15.3A STATISCAL MODEL

To characterize the UWB channel for applications HDR-WPAN three indoor channel models have been proposed: the Rayleigh tap delay line model (same as the one used in 802.11), the S-V [5] and the Δ-K [8] models. The S-V and Δ-K models use a Poisson statistical process in order to model the arrival time of multipath components (MPC). Nevertheless, the S-V model is unique in its approach of modeling the arrival time in cluster as well as rays within a cluster. In the modified S-V model, the channel impulse response (CIR), denoted by $h(t)$, is described as [9]

$$h(t)=\sum_{l=0}^{L_c-1}\sum_{k=0}^{L_r-1}\xi_l\beta_{k,l}e^{j\varphi_{k,l}}\delta\left(t-T_l-\tau_{k,l}\right), \tag{1}$$

where l represents the cluster index and k the ray index within a *l-th* cluster; L_c and L_r the number of clusters and rays, respectively; T_l the arrival time of the *l-th* cluster; and $\tau_{k,l}$ the arrival time of the *k-th* ray inside the *l-th* cluster. The inter-arrival time distribution of clusters and rays is given by an exponential distribution. ξ_l represents the amplitude of the *l-th* cluster, $\beta_{k,l}$ is the amplitude of the *k-th* ray inside the *l-th* cluster, and $\varphi_{k,l}$ is the phase of the ray inside the cluster. The average power of the *k-th* ray inside the *l-th* cluster is given by

$$\Omega_{k,l} = E\left[\left|\xi_l \beta_{k,l}\right|^2\right] = \Omega_0 \exp\left(-T_l/\eta\right)\exp\left(-\tau_{k,l}/\gamma\right), \quad (2)$$

where η and γ are the decay time constants of clusters and rays, respectively; and Ω_0 is the mean power of the first ray inside the first cluster. We assume that the amplitudes of the contributions $|\xi_l\beta_{k,l}|$ are mutually independent random variables (RV) and their phases $\varphi_{k,l}$ are uniformly distributed from 0 to 2π.

III. Frequency Domaing Channel Response

Hence, we will calculate the Fourier transform (FT) of the CIR given by (1). We will show that if the amplitude $|\xi_l\beta_{k,l}|$ of each UWB channel contribution is modeled as a log-normal RV and if the number of rays is high, the magnitude of the *i-th* subcarrier can be approximated by a Nakagami-*m* RV. The FT of the CIR given by (1) and denoted by $H(f)$ can be expressed as

$$H(f) = \sum_{l=0}^{L_c-1}\sum_{k=0}^{L_r-1} \xi_l \beta_{k,l} \exp\left[-j\left(2\pi f\left(T_l+\tau_{k,l}\right)-\varphi_{k,l}\right)\right]. \quad (3)$$

The amplitude of the *i-th* subcarrier in the frequency domain is $\alpha_i = |H(f_i)|$, where f_i is the *i-th* subcarrier frequency and it is modeled as a Nakagami-*m* [3] RV with probability density function (PDF) given by

$$f_{|H(f_i)|}\left(\alpha_i\right) = \frac{2}{\Gamma\left(m_{eq}^i\right)}\left(\frac{m_{eq}^i}{\Omega_{eq}^i}\right)^{m_{eq}^i} \alpha_i^{2m_{eq}^i-1} \exp\left(-\frac{m_{eq}^i \alpha_i^2}{\Omega_{eq}^i}\right), \quad (4)$$

where $\Gamma(\cdot)$ is the gamma function. Ω_{eq}^i the average power, and $m_{eq}^i \geq 0.5$ is the fading parameter of the *i-th* subcarrier. According to [3]

$$\Omega_{eq}^i \triangleq E\left[\left|H\left(f_i\right)\right|^2\right] = E\left[\left|H_R(f_i)\right|^2 + \left|H_I(f_i)\right|^2\right], \quad (5)$$

$$m_{eq}^i \triangleq \frac{E\left[\left|H(f_i)\right|^2\right]}{E\left[\left|H(f_i)\right|^4\right] - E\left[\left|H(f_i)\right|^2\right]}, \quad (6)$$

where $E[\cdot]$ denotes statistical expectation, $|H_R(f_i)|$ and $|H_I(f_i)|$ correspond to the magnitude of the real and imaginary parts of the *i-th* subcarrier, respectively.

IV. Numerical Results and Analysis

A. Average power and fading parameter.

The average power of the *i-th* subcarrier in frequency is obtained from (3) and (5), resulting

$$\Omega_{eq}^i = \sum_{l=0}^{L_c-1}\sum_{k=0}^{L_r-1}\Omega_{k,l} = \sum_{l=0}^{L_c-1}\sum_{k=0}^{L_r-1}\Omega_0 \exp\left[-\left(\frac{T_l}{\eta}+\frac{\tau_{k,l}}{\gamma}\right)\right], \quad (7)$$

The fading parameter of the *i-th* subcarrier, m_{eq}^i can be obtained from (5) and (6) as

$$m_{eq}^i = \frac{\left(\sum_{l=0}^{L_c-1}\sum_{k=0}^{L_r-1}\Omega_{k,l}\right)^2}{A\sum_{l=0}^{L_c-1}\sum_{k=0}^{L_r-1}\Omega_{k,l}^2 + \sum_{l=0}^{L_c-1}\sum_{k=0}^{L_r-1}\sum_{n=0}^{L_c-1}\sum_{m=0}^{L_r-1}\Omega_{m,n}\Omega_{k,l}}, \quad (8)$$

with, $A = \exp\left(4\sigma_{np}^2\right) - 2$ and σ_{np} the standard deviation of the log-normal fading [2], in nepers units, given by

$$\sigma_{np} = \frac{\ln(10)}{20}\sqrt{\sigma_c^2 + \sigma_r^2}, \quad (9)$$

where σ_c and σ_r are the standard deviations in dB units of clusters and rays, respectively. Fig 1 shows the comparison of the amplitude $|H(f_i)|$ PDF between the simulated data and the Nakagami-m approximation, where Ω_{eq} and m_{eq} are calculated from (7) and (8). 8 clusters and 12 rays by cluster were assumed in simulations [7]. The rest of parameters used in Fig.1 were: $\sigma_c = \sigma_r = 3.4$ dB, $\eta = 24$, $\gamma = 12$ and $\Omega_0 = 1$.

From Fig. 1, it can be observed that the Nakagami-*m* approximation and simulation curves are very similar and these results show that for a UWB channel with Nakagami-*m* fading and independents MPC: **a)** the magnitude of the channel response frequency at each frequency bin is approximately Nakagami-*m* distributed with the mean power given by (7) and the fading parameter given by (8); and **b)** these results also show that if the MPC number is higher that 96 (number of rays multiplied by number of clusters) then the relative error in the m_{eq} is less than 0.1% with respect to m_{eq}=1.

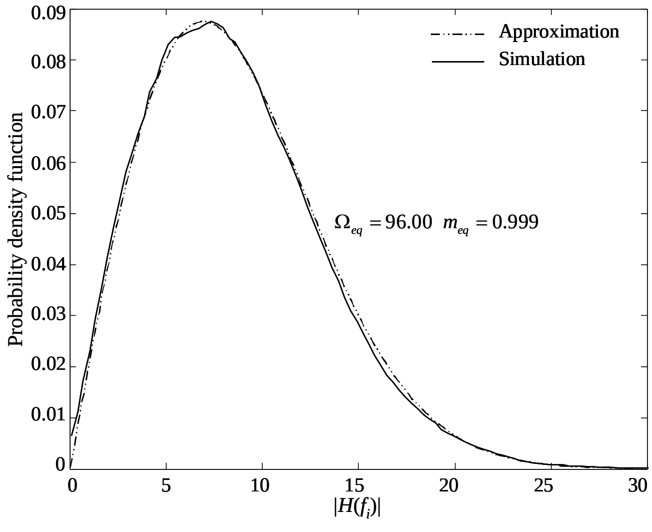

Figure 1. Probability density function of the channel frequency amplitude, $|H(fi)|$, using the Nakagami-m approximation

Fig. 2 shows the cumulative distribution function (CDF) for the amplitude $|H(f_i)$ of channel response frequency normalized by the mean power. Note that $|H(f_i)$ becomes Rayleigh for a sufficiently high number of MPC (typical environment in UWB channels). For instance, if the MPC number is higher than 63 contributions then the difference of the CDF for 10^{-3} between the simulated distribution and the Rayleigh distribution is less than 2 dB.

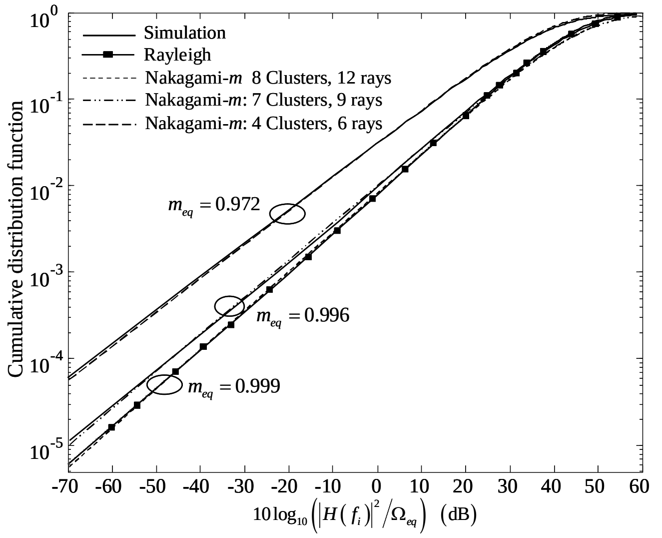

Figure 2. Cumulative distribution function of the normalized channel frequency amplitude, in dB units, for several UWB channels

B. Correlation coeffcient.

The correlation coefficient, $\rho_{i,j}$, between the *i-th* and the *j-th* subcarriers is obtained as

$$\rho_{ij} \triangleq \frac{\operatorname{cov}\left(\alpha_i^2,\alpha_j^2\right)}{\sqrt{\operatorname{var}\left(\alpha_i^2\right)\operatorname{var}\left(\alpha_j^2\right)}} = \frac{E\left[\alpha_i^2\alpha_j^2\right]-E\left[\alpha_i^2\right]E\left[\alpha_j^2\right]}{\sqrt{\operatorname{var}\left(\alpha_i^2\right)\operatorname{var}\left(\alpha_j^2\right)}}, \quad (10)$$

where var $(\cdot)$ is the variance of the RV given by [5]

$$\operatorname{var}\left(\alpha_i^2\right) = \operatorname{var}\left[\left|H(f_i)\right|^2\right] = \frac{\Omega_{eq}^2}{m_{eq}}. \quad (11)$$

Substituting (5) and (11) into (10), $\rho_{i,j}$ can be expressed as

$$\rho_{ij} = \left(\frac{E\left[\alpha_i^2\alpha_j^2\right]-\Omega_{eq}^2}{\Omega_{eq}^2}\right)m_{eq}. \quad (12)$$

From (7), (8) and (12), it can be easily obtained a closed-form expression of the correlation coefficient in the OFDM-UWB channel in the frequency domain given by

$$\rho_{ij} = \frac{A\sum_{l=0}^{L_c-1}\sum_{k=0}^{L_r-1}\Omega_{k,l}^2 + \sum_{l=0}^{L_c-1}\sum_{k=0}^{L_r-1}\sum_{n=0}^{L_c-1}\sum_{m=0}^{L_r-1}\Omega_{m,n}\Omega_{k,l}\cos B_{l,n}^{k,m}}{A\sum_{l=0}^{L_c-1}\sum_{k=0}^{L_r-1}\Omega_{k,l}^2 + \sum_{l=0}^{L_c-1}\sum_{k=0}^{L_r-1}\sum_{n=0}^{L_c-1}\sum_{m=0}^{L_r-1}\Omega_{m,n}\Omega_{k,l}}, \quad (13)$$

where $B_{l,n}^{k,m} = \frac{2\pi}{T_s}\left(\left(T_l+\tau_{k,l}\right)-\left(T_n+\tau_{m,n}\right)\right)(i-j)$. Fig. 3 shows the comparison of the correlation coefficient between simulated data and the analytical expression given by (13) for the following parameters: $\sigma_c = \sigma_r = 3.4$ dB, $\eta = 24$, $\gamma = 12$, $\Omega_0 = 1$, $L_c = 8$, and $L_r = 12$.

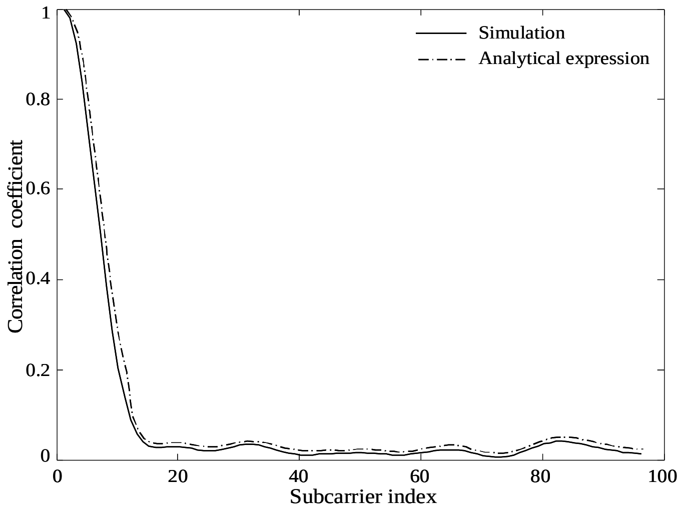

Figure 3. Correlation coefficient as a function of the subcarrier order with respect to the first subcarrier position

Fig. 4 shows the correlation coefficient as a function of the UWB channel delay spread, τ_{rms}, for four channel scenarios: CM1(τ_{rms} = 5.28 ns) CM2(τ_{rms} = 8.03 ns), CM3(τ_{rms} = 14.25 ns), and CM4(τ_{rms} = 25 ns). From this figure, we can observe a high dependence of the correlation coefficient between a couple of subcarriers on the delay spread. The parameters used in the simulations are given by [9].

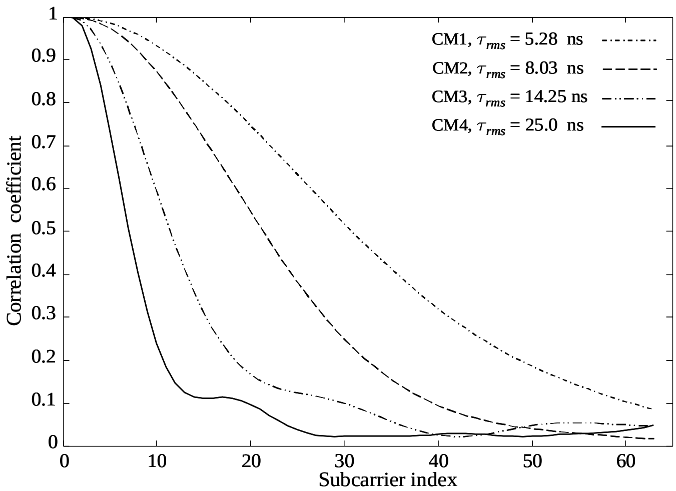

Figure 4. Correlation coefficient as a function of the subcarrier order

V. CONCLUSIONS

In this paper, we showed that UWB-OFDM channels with small-scale fading statistics modeled as log-normal RV can be approximated in the frequency domain by a Nakagami-*m* distribution, whose fading and mean power parameters are explicit functions of the delay parameters and decay time constants of the UWB channel. Moreover the subcarrier frequency distribution can be approximated by a Rayleigh distribution if the number of MPC is sufficiently high. Additionally, we found an exact expression for the correlation coefficient between a couple of subcarriers amplitudes in the frequency domain for the IEEE 802.15.3a UWB channel.

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