Cournot's model applied to cellphone service in Colombia, 1995-2001

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Cournot’s model applied to cellphone service in Colombia, 1995-2001

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Abstract

Purpose – The purpose of this paper is to develop and empirically test the conditions that describe adjustment velocities to reach equilibrium under Cournot’s duopoly model.

Design/methodology/approach – The paper uses a vector error correction (VEC) framework as the basis for determining and testing adjustment velocities using data about cellphone service in Colombia in the time period from 1995 to 2001.

Findings – Empirical evidence suggests the following: first of all, companies operating in the cellphone market behave as cournot’s competitors and have constant marginal costs; secondly, cellphone companies operating in the eastern zone of Colombia are in long-term equilibrium; and lastly, equilibrium adjustment velocities are statistically significant. As predicted by theory, in terms of welfare, the existence of equilibrium in Cournot’s model implies that cellphone users in the eastern zone of Colombia enjoy a small consumer surplus.

Originality/value – Testing the microeconomic implications of the equilibrium dynamics of Cournot’s model, using a VEC framework.

Keywords Telecommunication services, Service levels, Colombia

Paper type Research paper

1. Introduction

In 1994, the Colombian Government divided the country’s cellphone market into the eastern, western and coastal zones. In each of these zones, two cellphone providers were to compete with each other.

Market equilibria in each of the three zones are analyzed by means of Cournot’s model. Since, the series have unit roots, vector error correction (VEC) is used. The long-term equilibria and adjustment velocities are calculated by Johansen’s method. According to Fisher (1961), the use of adjustment velocities in Cournot’s model is justified since “... no seller is ever assumed to look once and for all at other outputs and then never to look again.”

The results show that the adjustment velocities are statistically significant and there are constant marginal costs for Comcel and Celumovil in the eastern zone.

This paper is divided into five parts. The second part analyzes the stability in each of the three zones. The third part discusses equilibrium in Cournot’s model through vector autoregression (VAR), and the VEC is derived. The fourth part provides an empirical analysis, and in the last part conclusions are presented.

The author would like to thank Jesus Otero, Boris Salazar and the anonymous referee for their valuable comments and discussions.
2. The Colombian cellphone market

Colombian cellphone service began in 1994 with a public bidding process. The objective was to establish two service providers in each zone so as to assure competitiveness, rapid growth and low prices to the consumer. The state-run monopoly in the telecommunications sector was thus brought to an end (Rumie, 2000).

Comcel and Celumóvil (BellSouth Oriente) were to compete in the eastern zone, while Celcaribe and Celumóvil de la Costa (BellSouth Costa) were to compete in the coastal zone. Lastly, Occel and Cocelco (BellSouth Occidente) were to compete in the western zone[1].

In order to explore the market behavior, we analyzed the market stability using the Heggestad and Rhoades (1976) instability index (II):

\[ II = \sum_{i=1}^{n} \left| \frac{S_{i,z,t+1} - S_{i,z,t}}{S_{i,z,t}} \right| \quad \forall i = \text{firm}, \quad z = \text{zone}, \quad t = \text{time} \quad (1) \]

Where \( S_{i,z,t} \) is the market share of company \( i \) in zone \( z \) at time \( t \). \( S_{i,z,t} \) is defined as the number of cellphone minutes sold by the firms in any given quarter. This index is developed from the absolute value of the change in market share of each service provider. It is used to indicate both positive and negative changes in market share, since both are indicative of market instability (Heggestad and Rhoades, 1976). The stability index in each zone is shown in Figure 1.

As you can see, the II in the eastern zone is above zero during the entire year. In some cases, firms take two or even three quarters to regain their position. The longest recovery time registered was from the first quarter of 1997 to the second quarter of 1999. Although the index has at times strayed from zero, in general, the firms have confirmed stability by maintaining their respective market shares. In the western zone, the II shows an increasingly unstable market since 1996. In the coastal zone, the II indicates that the firms do not maintain their market share throughout the year. In certain cases, firms need five quarters to recover market share (1996:1-1997:3), while in other cases, they have regained it within only three quarters (1997:3-1998:2).

![Instability Index](image)

Figure 1.
The value of 0.86 in the third quarter of 1998 is due to Celumovil Costa’s market share drop from 63 to 42 percent.

3. Stochastic version of Cournot’s duopoly model

Let \( q_i^t \) stand for the number of minutes of cellphone communications offered by service provider \( i \) during quarter \( t \). Let \( P(Q) \) stand for the inverse demand function \( [P(Q):\mathbb{R}_+ \rightarrow \mathbb{R}_+ \text{ is } C^2] \). Let us further assume that \( C(Q) \) is the cost function \( [C(Q):\mathbb{R}_+ \rightarrow \mathbb{R}_+ \text{ is } C^2] \) and \( P(0) > C(0) \). The cost function is
\[
\begin{align*}
\text{cost function for the } i \text{th firm} & = g_i + c_i q_i + 1/2 d q_i^2. \end{align*}
\]
Where \( g_i \geq 0 \) and if \( d > 0 \), there are increasing marginal costs; if \( d < 0 \), then there are decreasing marginal costs; and if \( d = 0 \), there are constant marginal costs. Each seller expects that the output of every other seller will not change from \( t - 1 \) to \( t \). On this assumption, he expects \( p_i \) to be a function of this output alone in the \( t \) period (Fisher, 1961). Then the price is
\[
\begin{align*}
p_i^t = a - bq_i^t - bq_{i-1}^t. \end{align*}
\]
The profit function is \( \prod_t(q_i^t, q_j^t) = q_i^t[a - bq_i^t - bq_{i-1}^t] - g - c_i q_i - 1/2dq_i^2 \). And the profit maximization under incomplete information is:
\[
\begin{align*}
\frac{\partial \Pi_i^t}{\partial q_i^t} &= a - 2bq_i^t - bq_{i-1}^t - c_i - dq_i^2 + \varepsilon_i^t, \\
\frac{\partial \Pi_i^t}{\partial q_{i-1}^t} &= a - 2bq_{i-1}^t - bq_i^t - c_i - dq_i^2 + \varepsilon_i^t.
\end{align*}
\]
Where \( \varepsilon_i^t \) is a random variable that shows an aspect of incomplete information of the cost function for the \( i \)-firm or demand uncertainty (Dutta, 1999). The reaction function in matrix form is:
\[
\begin{align*}
\begin{bmatrix} q_i^t \\ q_{i-1}^t \\
\end{bmatrix} &= \begin{bmatrix} \frac{a-c_i}{2b+d} \\ \frac{a-c_{i-1}}{2b+d} \\
\end{bmatrix} + \begin{bmatrix} 0 & -\frac{b}{2b+d} \\ -\frac{b}{2b+d} & 0 \\
\end{bmatrix} \begin{bmatrix} q_i^{t-1} \\ q_{i-1}^{t-1} \\
\end{bmatrix} + \begin{bmatrix} \varepsilon_i^t \\ \varepsilon_{i-1}^t \\
\end{bmatrix}.
\end{align*}
\]
The reaction function in Cournot’s duopoly model is also expressed as:
\[
\begin{align*}
\begin{bmatrix} q_i^t \\ q_{i-1}^t \\
\end{bmatrix} &= \begin{bmatrix} \omega_{11} & \Omega_{12} \\ \omega_{21} & \Omega_{21} \\
\end{bmatrix} \begin{bmatrix} q_i^{t-1} \\ q_{i-1}^{t-1} \\
\end{bmatrix} + \begin{bmatrix} \varepsilon_i^t \\ \varepsilon_{i-1}^t \\
\end{bmatrix}.
\end{align*}
\]
where:
\[
\begin{align*}
\omega_{11} &= a - c_i \quad ; \quad \omega_{21} = a - c_{i-1} \\ \Omega_{12} &= \frac{b}{2b+d} \\ \Omega_{21} &= \frac{b}{2b+d}.
\end{align*}
\]
Rearranging equation (4), we obtain the following VAR:
\[
\begin{align*}
Q_i = \omega + \Omega Q_{i-1} + \varepsilon_i; \quad \varepsilon_i \sim i.i.d(0, \Sigma)
\end{align*}
\]
The VAR in equation (5) has a single equilibrium point if the characteristic roots lie within a unit circle, which is in fact true as long as \( \{q_i^1\} \) and \( \{q_i^2\} \) are \( l(0) \). If we further assume that \( \{q_i^1\} \) and \( \{q_i^2\} \) are \( l(1) \), then the VAR will be represented by the following VEC:
\( \Delta Q_t = \Pi Q_{t-1} + \epsilon_t \)  \( (6) \)

Where \( \Pi = -(I - \Omega) \) is a 2 × 2 matrix. If we let \( r \) be the range of \( \Pi \) then \( r \) is also the number of co-integration vectors. Thus, if \( r = 1 \), there is but one co-integration vector, and each sequence of \( \{q_i^t\} \) can be described in terms of error corrections. By normalizing with respect to \( q_1^t \) and letting \( a_1 = \Pi_{11} \beta_1 = \Pi_{12}/\Pi_{11} \) one obtains:

\[
\Delta q_1^t = \alpha_1(q_{1i-1}^1 + \beta_1 q_{2i-1}^2) + \epsilon_{1t}
\]

\[
\Delta q_2^t = \alpha_2(q_{1i-1}^1 + \beta_1 q_{2i-1}^2) + \epsilon_{2t}
\]

In the long run, the following conditions must be met:

\[
q_{1i-1}^1 - \beta_1 q_{2i-1}^2 = 0; \quad (q_{1i}^1, q_{2i}^2) - CI(1, 1)
\]

and \([1 \quad \beta_1]\) being the normalized co-integration vector. On the other hand, if there are constant marginal costs, then \( \beta_1 = -0.5 \). Observe that \( \beta_1 = -b/(2b + d) \) and if \( d = 0 \) (constant marginal cost) then \( \beta_1 = -0.5[3] \).

Following Johansen’s (1988, 1991, 1995) method, if the range (\( r \)) of matrix \( \Pi \) is equal to \( r < n \) (\( n \) being the number of variables), then matrix \( \Pi \) can be written as:

\[
\Pi = \alpha\beta'
\]

where \( \beta' \) is the co-integration vector and \( \alpha \) is the resultant vector of the adjustment velocities that ensure long-term equilibrium. With regard to these adjustment velocities, we bring on the work of Fisher (1961):

… all discussions of Cournot’s model allow sellers to receive new information about the outputs of their rivals. No seller is ever assumed to look once and for all at other outputs and then never to look again. Rather, he is assumed to look, then adjust, then look again, and so forth.

Thus, adjustment velocities tend towards equilibrium. That is, “… they are the fractions of the distance from the actual to desired output which are covered in one time period” (Fisher, 1961). When \( \alpha_i = 1 \), the adjustment is complete – as indicated by Theocharis (1960). Therefore, if \( \alpha_i = 1 \), then 100 percent of the disequilibrium is corrected within a single period.

### 4. Empirical analysis

The empirical analysis of Cournot’s model, as represented by equation (6), begins with a unit root test of all variables. The data – from the fourth quarter of 1995 to the second quarter of 2001 – were provided by the Colombian Ministry of Communications. Since, we are dealing with a series of quarters, we use the HEGY test (Hylleberg et al., 1990) to compare unit root, \( I(1) \), at zero frequency, at a semiannual frequency and at an annual frequency. The method is as follows:

\[
\Delta_4 Y_t = \alpha + \gamma_1 Y_{1,t-1} - \gamma_2 Y_{2,t-1} + \gamma_3 Y_{3,t-1} + \gamma_4 Y_{3,t-2} + \epsilon_t
\]

where

\[
Y_{1,t-1} = (1 + L + L^2 + L^3)Y_{t-1}, \quad Y_{2,t-1} = (1 - L + L^2 - L^3)Y_{t-1}, \quad Y_{3,t-1} = (1 - L^2)Y_{t-1}.
\]

Then test the following hypothesis:
\[ H_0: \gamma_1 = 0 \quad I(1) \text{ at zero frequency.} \]
\[ H_0: \gamma_2 = 0 \quad I(1) \text{ at a semiannual frequency.} \]
\[ H_0: \gamma_3 = \gamma_4 \quad I(1) \text{ at an annual frequency.} \]

The results are shown in Table I.

From Table I, we conclude that the series in the eastern zone are integrated as to order and frequency. In the western zone, the series are integrated at zero frequency. Additionally, the Occocc series are integrated at semiannual frequency. In the coastal zone, the series are integrated at different frequencies. Thus, if there is any co-integration it can only exist in the eastern zone. That is, only in the eastern zone there can be a stable relationship among the variables – and hence the possibility of long-term equilibrium – given the integration of variables as to both order and frequency, which is the requirement for co-integration[4].

Following Johansen’s method, \( \lambda \text{trace} \) and maximum-eigenvalue was estimated so as to determine the number of cointegration vectors in the eastern zone[5].

From Tables II and III, we can establish the existence of a co-integration vector \( (r = 1)[6,7] \). Subsequently, tests were conducted for long-term exclusion and to ascertain to what degree the variables were stationary and weakly exogenous.

From Table IV we conclude that no variable may be excluded unless it is both stationary and weakly exogenous. Upon selecting the co-integration vector \( (r = 1) \), and normalizing with respect to \( q_i^{celori} \) we obtain an estimate of equation (9)[8,9].

From Table V, we conclude that Celumovil corrects 22 percent of the disequilibrium in its cellphone minutes within one quarter, while Comcel corrects 34 percent. Since, at a 95 percent confidence level and with only 21 data points, the \( t \)-statistic yields 1.72, we

<table>
<thead>
<tr>
<th>Variables/( \gamma )</th>
<th>( \gamma_1 )</th>
<th>( \gamma_2 )</th>
<th>( \gamma_3 = \gamma_4 )</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Celori (Celumovil oriental)</td>
<td>-2.1329</td>
<td>2.93609</td>
<td>6.2416</td>
<td>( I(1) ) at zero-frequency</td>
</tr>
<tr>
<td>Comori (Comcel)</td>
<td>-1.04668</td>
<td>3.18738</td>
<td>4.45</td>
<td>( I(1) ) at zero-frequency</td>
</tr>
<tr>
<td>Celocc (Celumovil Occidente)</td>
<td>-0.54801</td>
<td>2.25504</td>
<td>5.6774</td>
<td>( I(1) ) at zero-frequency</td>
</tr>
<tr>
<td>Occocc (Occel)</td>
<td>0.02761</td>
<td>1.49434</td>
<td>6.63954</td>
<td>( I(1) ) at zero and semiannual frequency</td>
</tr>
<tr>
<td>Celcosta (Celumovil Costa)</td>
<td>-1.28600</td>
<td>3.14013</td>
<td>4.24616</td>
<td>( I(1) ) at zero-frequency</td>
</tr>
<tr>
<td>Caribecosta (Celcaribe)</td>
<td>-3.20664</td>
<td>1.56834</td>
<td>8.06550</td>
<td>( I(1) ) at semiannual frequency</td>
</tr>
<tr>
<td>Critical value ( N = 25 )</td>
<td>2.77</td>
<td>1.76</td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>Critical value ( N = 30 )</td>
<td>2.78</td>
<td>1.79</td>
<td>3.82</td>
<td></td>
</tr>
</tbody>
</table>

Source: The critical values at the 95 percent confidence level are reported in Charemza and Deadman (1997, Table 8)

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>Alternative hypothesis</th>
<th>( \lambda \text{trace} )</th>
<th>90 percent</th>
<th>95 percent</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r = 0 )</td>
<td>( r &gt; 0 )</td>
<td>16.66</td>
<td>13.308</td>
<td>15.340</td>
<td>Reject the null</td>
</tr>
<tr>
<td>( r \leq 1 )</td>
<td>( r &gt; 1 )</td>
<td>2.78</td>
<td>2.706</td>
<td>3.841</td>
<td>Do not reject the null</td>
</tr>
</tbody>
</table>

Source: Critical values from Hansen and Juselios (1995)

<table>
<thead>
<tr>
<th>Table I.</th>
<th>Unit root test, ( N = 23 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table II.</td>
<td>Number of cointegrating vectors, ( N = 21 )</td>
</tr>
</tbody>
</table>
may conclude that the adjustment velocities are statistically significant, and that Comcel is faster when adjusting to equilibrium.

Below, we prove the hypothesis that there are constant marginal costs. From equation (9), we find that once $q_{\text{Celori}}^t$ is normalized, the hypothesis of constant marginal costs should be $\beta' = [1 - 0.5]$. The value given by the Lr-test is 0.45, with a $p$-value of 0.5[10]. Hence, at a 5 percent significance level, we cannot reject the hypothesis of constant marginal costs.

5. Conclusions

When one analyzes the market stability of cellphone service within the three zones into which the Colombian Government divided the market, one finds a high level of stability in the eastern zone in the 1996-2001 period. Stability in this zone is achieved in just two or three quarters, although at times it has taken longer (as occurred from the first quarter of 1997 to the fourth quarter of 1998). The western zone has been increasingly unstable since 1996. As for the coastal zone, in the second quarter of 1998 the II has shown an increasing tendency to stray from zero.

The relative market stability (or instability) must be taken into consideration in analyzing the equilibrium conditions of each zone. In fact, the only zone that has equilibrium between the competing firms is also the zone with highest market stability – the eastern zone. Since, market stability (or instability) results from the behavior of individual firms, the connection with equilibrium in Cournot’s model is intuitively logical.

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>Alternative hypothesis</th>
<th>Maximum-eigenvalue</th>
<th>95 percent</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = 0$</td>
<td>$r &gt; 0$</td>
<td>24.34893</td>
<td>19.38704</td>
<td>Reject the null</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>$r &gt; 1$</td>
<td>5.284625</td>
<td>12.51798</td>
<td>Do not reject the null</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$r$</th>
<th>g.l.</th>
<th>$\chi^2$</th>
<th>Celori</th>
<th>Comori</th>
<th>Test</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long-run exclusion</td>
<td>1</td>
<td>1</td>
<td>3.84</td>
<td>9.06</td>
<td>10.40</td>
<td>$H_0: \beta_1 = 0$</td>
</tr>
<tr>
<td>Stationarity</td>
<td>1</td>
<td>1</td>
<td>3.84</td>
<td>10.40</td>
<td>9.06</td>
<td>$H_0: \beta = (H_{i\phi})$</td>
</tr>
<tr>
<td>Weak exogeneity</td>
<td>1</td>
<td>1</td>
<td>3.84</td>
<td>4.05</td>
<td>7.61</td>
<td>$H_0: \alpha_1 = 0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>Lag</th>
<th>$\beta' = [\beta_0 \beta_1]$</th>
<th>$\alpha' = [\alpha_{\text{Celori}} \alpha_{\text{Comori}}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${q^t_{\text{Celori}}, q^t_{\text{Comori}}}$</td>
<td>1</td>
<td>$1 - 0.579$</td>
<td>0.221</td>
</tr>
</tbody>
</table>

Note: $t$-statistic in parentheses
In particular, the results of Cournot’s model, as applied to the eastern zone, indicate that the model should take the form of a VEC. This is because there is a cointegration vector and the adjustment velocities are statistically significant. The results also indicate that the VEC may be restricted by \((1, -0.5)\) and show the existence of constant marginal costs. The results also indicate that while Celumóvil corrects 22 percent of the disequilibrium in its cellphone minutes within one quarter, Comcel corrects 34 percent. This result is of importance to us because the statistical significance of the adjustment velocities – being a stabilizing factor – reflects constant marginal costs within the cellphone service providers in the eastern zone.

In terms of welfare, the existence of equilibrium in Cournot’s model implies that cellphone customers in the eastern zone enjoy a small consumer surplus. In Cournot’s duopoly, with positive costs for both firms and inverse demand functions \(p'(Q) < 0 \forall Q > 0\), the market price is above the perfect competition price, yet lower than a monopoly price (Mas-Colell et al., 1995). See Hann and Maks (1996) for a comprehensive discussion of price and welfare in Cournot’s model with constant marginal costs.

Notes
1. As of March 15, 2001, Celumóvil and Cocelco changed their names to BellSouth, following the purchase of these firms by BellSouth in 2000.
2. Given that \(E(\sigma) = 0\), system (5) is similar to equation (2.6) of Fisher (1961).
3. As suggested by the referee, this kind of hypothesis fails to include other variables such as capacity, cost efficiency, and asset parsimony dimension which could impact costs. However, in order to rule out any other kinds of marginal costs, it is essential that this hypothesis \((d = 0)\) holds true.
4. There can be no VEC in the coastal zone, since the series are not I(1) at the same frequency (Hylleberg et al., 1990). In the western zone, given that Celocc is I(1) at zero frequency and Occecc is I(1) at both zero and semiannual frequency, co-integration could occur. Hylleberg et al. (1990) suggest the use of applied filters just to remove seasonal roots, and then checking for cointegration. When this is done, with X-11 filter, the \(Lr\)-test produces 14.243 and 0.050. Thus, under no circumstances can \(H_0\) be rejected because the series are not co-integrated. However, Otero and Smith (1999) show that seasonal filters reduce the power of the co-integration test. Nonetheless, once VEC is calculated without seasonal adjustment to the series, the adjustment velocities are not significant.
5. For a detailed analysis of Johansen’s method, see the CATS in RATS manual by Hansen and Juselios (1995), Charemza and Deadman (1997), Enders (1995) and Davidson (2000). The cointegration technique has been applied in different topics by different authors such as Love and Chandra (2005), Agbola and Damoense (2005), Bahmani-Oskooee and Miteza (2004), Zhen Zhu (2001), Bahmani-Oskooee (1999), Als and Bahmani-Oskooee (1995), King (1993) and Simmons (1992).
6. The corrections of Cheung and Lai (1993) with regard to critical values for \(r = 0\) at 95 and 90 percent confidence levels are, respectively, 16.79 and 14.70. This, however, is not the case. “… Proper corrections of the critical values in finite samples are therefore particularly essential when the estimated system contains many variables and/or long lags,” as the aforementioned authors themselves point out.
7. Note that when \(r = 2\), all the variables in the VAR are I(0). This contradicts the results of Table I Charemza and Deadman (1997).
8. The VEC was estimated by means of the CATS drift option. Additionally, VEC-centered seasonal dummies were employed.

9. The tests indicate that the model is free of problems. The residuals are normal and the eigenvalues are within the unit circle. There are neither auto-correlation nor ARCH effects.

10. The test is of type $H_0: \beta = H_0$. The test employs the statistical $Lr$-test, which follows a $\chi^2(1)$ distribution.

References


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